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Modeling of Power Law Fluid for Flow through Artery with Aneurysm

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Abstract
The power law model has been modified to be suitable for analysis of the steady flow of blood passing through rigid cylindrical artery with an axially symmetric aneurysm. The results show that as the radius of aneurysm increases, the pressure drops across an aneurysm length for power law decrease. This decrease may indicate that there is back flow occurring in the aneurysm length.

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Introduction

The mathematical model is general concept of collaborations among mathematics and other science. This collaboration is done in many disciplines like mathematical botany, mathematical zoology, mathematical ecology, mathematical genetics, mathematical physiology, and mathematical bio sciences. In the field of mathematical bio sciences, many mathematical models are used to study hemodynamic parameters; specially, pressure, shear stress, and viscosity. One of these models is power low model which can be described as T= $\mu(dv/dr)^n$, where T is shear stress, (dv/dr) is shear rate, n is related to the behavior of fluid flow and μ is the viscosity[1].

The power low model deals with Newtonian flow when n=1, and with non-Newtonian when n≠1. In Newtonian flow, the sloop of shear stress and shear rate is a straight line, and the coefficient of viscosity is constant. While in non – Newtonian flow, the relationship between shear stress and shear rate is a curve, and the viscosity is not constant [2].

Aneurysm is an enlarged size of arteries caused by a weakening of the arteries wall [3][4]. The relation between the aneurysm and hemodynamic of blood flow is considered the interest subject of researchers [5][6]. Existence of an aneurysm in the artery can disrupt the normal functioning of the circulatory system; in particular, it may affect the pressure drop and resistance of flow [7].

This work aims to modify the power law model to be suitable for analysing the blood flow through large arteries in the presence of aneurysm.

Modifying the power law model.

We consider the flow of blood flow, passing through rigid and cylindrical artery with an axially symmetric aneurysm of radii R and R(z), behaving as a power law fluid. The geometric of the assuming artery and its coordinates system is given in equation (1) and figure (1).

$$R(z) = \begin{cases} R+\frac{h}{2}+\frac{h}{2}COS(\pi\frac{z}{z_0})), & |z|<z_0, \\ R,,,|z|\geq z_0 \end{cases} \qquad (1)$$

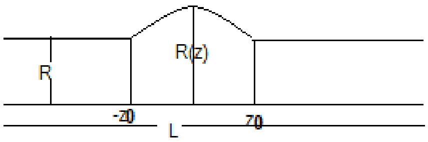

Figure 1: the geometry and its coordinates of aneurysm artery

Then based on the conditions mentioned above the axial velocity can be expressed as

$$\frac{1}{r}\frac{\partial}{\partial r} r\mu(\frac{\partial v}{\partial r})^{n} = \frac{\partial p}{\partial z} \qquad (2)$$

$|Z| \geq Z_0$, r=R then v=0 at $|Z| \leq Z_0$, r=R(z) then v=0 (3)

Integrating equation (2), we find

$$r\mu(\frac{\partial v}{\partial r})^{n} = \frac{\partial p}{2\partial z} r^{2} + c$$

$$r(\frac{\partial v}{\partial r})^{n} = \frac{\partial p}{2\mu \partial z} r^{2} + \frac{c}{\mu}$$

$$r(\frac{\partial v}{\partial r})^{n} = \tau . r + \frac{c}{\mu}$$

Where τ is the wall shear stress and μ is the viscosity of blood

But c=0 because, on the axis, ∂v/∂r =0, and the wall shear stress (**τ**) is finite.

Then $\frac{\partial v}{\partial r} = \tau^{1/n} r^{1/n}$ is the power law model and can be written in the form of

$$\frac{\partial v}{\partial r} = (\frac{\partial p}{2\mu . \partial z})^{1/n} r^{1/n} \qquad (4)$$

Integrating equation (4) with respect to r and use equation (3), we get

$$v = \frac{n}{n+1}(\frac{\partial P}{2\mu \partial z})^{1/n}(R(z)^{1/n+1}) - r^{1/n+1}) \qquad (5)$$

Equation (5) is the equation of velocity (v) of power law in the prescience of aneurysm.

The flow rate Q can be expressed as

$$Q = \int_{0}^{R(z)} 2\pi r v dv \quad . \qquad (6)$$

Integrating equation (6) by prate and use equation (5)

$$Q = \pi v r^{2} \Big|_{0}^{R(z)} - \int_{0}^{R(z)} r^{2}\pi dv = 0 - \pi \int_{0}^{R(z)} r^{2} dv$$

$$Q = \frac{\pi n}{(n+1)}(\frac{\partial p}{2\mu \partial z})^{1/n} \int_{0}^{R(z)} r^{2} \frac{(n+1) r^{1/n}}{n} dr)$$

$$Q = \pi(\frac{\partial p}{2\mu \partial z})^{1/n} \int_{0}^{R(z)} r^{(1/n)+2} dr$$

$$Q = \frac{\pi n}{(3n+1)}\left(\frac{\partial p}{2\mu\partial z}\right)^{1/n} R(z)^{(1/n)+3}. \qquad (7)$$

Equation (7) represents the flow rate (Q) of power law fluid. Solving equation (7) for (Əp/Əz)

$$\boxed{\frac{\partial p}{\partial z} = \frac{2\mu(3n+1)^n Q^n}{(\pi n)^n}\frac{1}{(R(z)^{1/n+3})^n}} \qquad (8)$$

Equation (8) is the equation of pressure gradient (Əp/Əz)

Integrating equation (8), with respect to z, to find the equation of pressure drop (Δp).

$$\Delta p = \frac{2\mu(3n+1)^n Q^n}{(\pi n)^n R^4}\int_{-z_0}^{z_0}\frac{dz}{1+\frac{R}{2}(1+\cos\frac{\pi z}{z_0})^{(1+3n)}} \qquad (9)$$

Equation (9) is the equation of pressure drop (Δp), across the length of aneurysm part only, for power law fluid.

Analysis for power law

The power law model deals with Newtonian flow when n=1, and with non-Newtonian when n≠1.

Put n=1 in equation (9) and integrating the same, we get

$$\Delta p = \frac{16z_0\mu Q(1+\frac{3h}{2R}+\frac{9}{8}(\frac{h}{R})^2+\frac{5}{16}(\frac{h}{R})^3}{\pi R^4((1+\frac{h}{R}+\frac{1}{2}(\frac{h}{R})^2)^{\frac{7}{2}}} \qquad (10)$$

Equation (10) is the equation of pressure drop, across the length ($2z_0$) of aneurysm part only, for Newtonian fluid.

The pressure drop across the whole length of artery can be expressed as

(The pressure drop along the full artery) + (the pressure drop across the length of aneurysm part or whenh≠0 – the pressure drop across normal part along the aneurysm or when h=0)

$$\Delta p = \frac{8L\mu Q}{\pi R^4} + \frac{16z_0\mu Q(1+\frac{3h}{2R}+\frac{9}{8}(\frac{h}{R})^2+\frac{5}{16}(\frac{h}{R})^3}{\pi R^4((1+\frac{h}{R}+\frac{1}{2}(\frac{h}{R})^2)^{\frac{7}{2}}} - \frac{16z_0\mu Q}{\pi R^4}$$

$$\Delta p = \frac{8L\mu Q}{\pi R^4} + \frac{16 z_0 \mu Q}{\pi R^4}\left(\frac{(1+\frac{3h}{2R}+\frac{9}{8}(\frac{h}{R})^2+\frac{5}{16}(\frac{h}{R})^3}{((1+\frac{h}{R}+\frac{1}{2}(\frac{h}{R})^2)^{\frac{7}{2}}} - 1\right) \qquad (11)$$

Equation (11) is the equation of pressure drop (Δp) across whole length (L) of artery with aneurysm of length $2z_0$.

At h=0 equation (11) is reduced to $(\Delta p)_p = \frac{8L\mu Q}{\pi R^4}$ which is Poiseuille's fluid.

$$\frac{(\Delta p)}{(\Delta p)p} = \int_{-z_0}^{z_0} \frac{dz}{1+\frac{R}{2}(1+\cos\frac{\pi z}{z_0})^{(1+3n)}} \qquad (12)$$

Equation (12) represents the distribution of pressure drop across the length of aneurysm, for power law fluid, with respect to Poiseuille's fluid.

When n=1, equation (12) becomes

$$\frac{\Delta p}{(\Delta p)_p} = \left(\frac{(1+\frac{3h}{2R}+\frac{9}{8}(\frac{h}{R})^2+\frac{5}{16}(\frac{h}{R})^3}{((1+\frac{h}{R}+\frac{1}{2}(\frac{h}{R})^2)^{\frac{7}{2}}}\right)$$

When n=2/3, equation (12) becomes

$$\frac{\Delta p}{(\Delta p)_p} = \left(\frac{(1+\frac{h}{2R})}{((1+\frac{h}{R}+2(\frac{h}{R})^2))^{\frac{3}{2}}}\right)$$

The Results and discussion

Equation (11) is the equation of pressure drops across the length of artery in the presence of aneurysm. This equation is solved by using mathematical software called Math 3.0 for the given boundary values of lengths, radii, flow rate and number of arteries. The results listed in table 1 and table 2 show that as the radius of aneurysm increases, the pressure drop decreases. It's found that the pressure drops for aorta and for large arteries at h=0 were 33.4 dynes/cm^2 and 1910 dynes/cm^2 respectively. These results have a very good agreement with the results estimated by (Irving 2008). This encourages us to use this model to calculate the pressure drop across the whole length of artery in the presence of aneurysm, h≠0.

Also, from figure 1, its clear that as h/R increase, the ratio $\Delta p/(\Delta p)_p$ decreases by a smaller amount in case n<1 than it does for n=1.

Table 1: pressure drop of large artery for R=0.2 cm, l=75cm, Q=(80/200) cm^3/s

h (cm)	Pressure drop (dyens/cm^2)
0	1909.859
0.022222	1888.038

0.044444	1869.166
0.066667	1854.024
0.088889	1842.410
0.111111	1833.727
0.133333	1827.317
0.155556	1822.606
0.177778	1819.14
0.2	1816.576

Table 2: pressure drop of Aorta for R=1.25 cm, l=10cm, Q= 80cm^3/s

h (cm)	Pressure drop (dyens/cm^2)
0	33.37721
0.138889	30.51708
0.277778	28.0434
0.416667	26.05876
0.555556	24.53653
0.694444	23.39841
0.833333	22.55828
0.972222	21.94079
1.111111	21.48639
1.25	21.15043

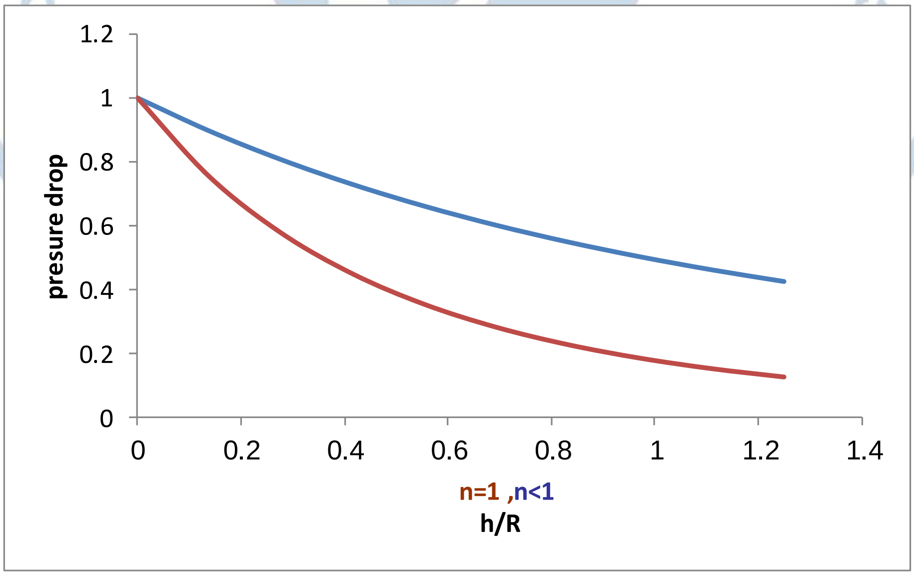

Figure 2: the distribution of pressure drop across an aneurysm when n=1and n<1

Conclusion

In this study, the power law model has been modified under the conditions mentioned in section 2. This model can be used in case of single flow, one layer, with one viscosity but could not be used in case of couple flow or two layers.

If we put n=1 and h=0, we get results of special case of Poiseuille flow that is clear from equation (11) and also from table 1 and table 2 where the pressure drop for aorta and for large arteries are 33.4 dynes/cm^2 and 1910 dynes/cm^2 respectively

If we put n=1 and h≠0, we get results of special case of Newton flow. In this case, the results show decreasing of pressure drop. This decrease may indicate that there is back flow occurring in the aneurysm length.

This model also can be used as empirical law, in experimental studies, to estimate the viscosity of the blood that flow through different sizes of aneurysm pipe and compare the same to each other's as well as to compare the viscosity of the blood flow in normal pipe with the viscosity of the blood flow through different sizes of aneurysm pipe.

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نموذج قانون الطاقة، تمدد الأوعية الدموية، ضغط التدفق

الملخص
تم تطوير قانون القوة ليكون مناسب في تحليل تدفق الدم المار خلال شريان أسطواني صلب منتفخ ومتماثل حول محور التدفق الثابت. وتبين النتائج ان الزيادة في قطر منطقة الانتفاخ (زيادة حجم المنطقة المنتفخة) يودي الى نقصان في فرق الضغط على طول الشريان. هذا النقصان في الضغط قد يكون مؤشر على حدوث تدفق عكسي للدم في منطقة الانتفاخ.